SEQUENCES AND SERIES — *Answers*

1 $= 1 + 4(4x) + 6(4x)^2 + 4(4x)^3 + (4x)^4$

$= 1 + 16x + 96x^2 + 256x^3 + 256x^4$

2 **a** $u_5 = 3 \times (-2)^4 = 48$

b $S_{10} = \frac{3[1-(-2)^{10}]}{1-(-2)} = -1023$

c positive terms form GP:
$a = 3,\ r = (-2)^2 = 4$

$S_8 = \frac{3(4^8-1)}{4-1} = 65\,535$

3 **a** $= 1 + 7(3x) + \frac{7\times6}{2}(3x)^2 + \frac{7\times6\times5}{3\times2}(3x)^3 + \frac{7\times6\times5\times4}{4\times3\times2}(3x)^4 + \ldots$

$= 1 + 21x + 189x^2 + 945x^3 + 2835x^4 + \ldots$

b let $x = 0.01$

$1.03^7 \approx 1 + 0.21 + 0.0189 + 0.000\,945 + 0.000\,028\,35$

$= 1.229\,87$ (5dp)

4 GP: $a = 8,\ r = 2,\ n = 10$

$S_{10} = \frac{8(2^{10}-1)}{2-1} = 8184$

5 **a** $= 2^5 + 5(2^4)x + 10(2^3)x^2 + 10(2^2)x^3 + 5(2)x^4 + x^5$

$= 32 + 80x + 80x^2 + 40x^3 + 10x^4 + x^5$

b $= 32 - 80x + 80x^2 - 40x^3 + 10x^4 - x^5$

c $(2 + \sqrt{5})^5 = 32 + 80(\sqrt{5}) + 80(\sqrt{5})^2 + 40(\sqrt{5})^3 + 10(\sqrt{5})^4 + (\sqrt{5})^5$

$= 32 + 80\sqrt{5} + 400 + 200\sqrt{5} + 250 + 25\sqrt{5}$

$= 682 + 305\sqrt{5}$

$\therefore (2 + \sqrt{5})^5 - (2 - \sqrt{5})^5$

$= (682 + 305\sqrt{5}) - (682 - 305\sqrt{5})$

$= 610\sqrt{5},\ k = 610$

6 **a** amount in account after 3rd payment in

$= 200 + (1.005 \times 200) + (1.005^2 \times 200)$

$= 603.005$

interest paid at end of 3rd month

$= 0.005\times603.005 = £3.02$ (nearest penny)

b amount paid in $= 12 \times 200 = £2400$

amount in account after 12 months

$= 200(1.005 + 1.005^2 + \ldots + 1.005^{12})$

$= 200 \times S_{12}$ [GP: $a = 1.005,\ r = 1.005$]

$= 200 \times \frac{1.005(1.005^{12}-1)}{1.005-1} = 2479.45$

total interest $= 2479.45 - 2400 = £79.45$

7 $= 1 + 8(-3x) + \frac{8\times7}{2}(-3x)^2 + \frac{8\times7\times6}{3\times2}(-3x)^3 + \ldots$

$= 1 - 24x + 252x^2 - 1512x^3 + \ldots$

8 **a** $S_n = a + ar + ar^2 + \ldots + ar^{n-1}$

$rS_n = ar + ar^2 + \ldots + ar^{n-1} + ar^n$

subtracting, $S_n - rS_n = a - ar^n$

$S_n(1 - r) = a(1 - r^n)$

$S_n = \frac{a(1-r^n)}{1-r}$

b $r = 6 \div 3 = 2$

$a \times 2^3 = 3 \ \therefore a = \frac{3}{8}$

$S_{16} = \frac{\frac{3}{8}(2^{16}-1)}{2-1} = 24\,575\frac{5}{8}$

9 **a** $= 1 + n(ax) + \frac{n(n-1)}{2}(ax)^2 + \ldots$

$= 1 + anx + \frac{1}{2}a^2n(n-1)x^2 + \ldots$

b $\frac{1}{2}a^2n(n-1) = 3an$

$a^2n(n-1) = 6an$

$an[a(n-1) - 6] = 0$

$n \neq 0 \;\therefore\; a(n-1) - 6 = 0$

$an - a = 6$

$n = \frac{6+a}{a}$

c $n = 10 \;\therefore$ coeff. of $x^3 = \frac{10\times9\times8}{3\times2} \times (\frac{2}{3})^3 = 35\frac{5}{9}$

10 $= 2^6 + 6(2^5)(5x) + \frac{6\times5}{2}(2^4)(5x)^2 + \ldots$

$= 64 + 960x + 6000x^2 + \ldots$

11 **a** $\frac{162}{1-r} = 486$

$1 - r = \frac{162}{486} = \frac{1}{3} \quad \therefore\; r = \frac{2}{3}$

b $u_6 = 162 \times (\frac{2}{3})^5 = \frac{64}{3}$ or $21\frac{1}{3}$

c $S_{10} = \frac{162[1-(\frac{2}{3})^{10}]}{1-\frac{2}{3}} = 477.572$

12 **a** $= 1 + 4(3x) + 6(3x)^2 + 4(3x)^3 + (3x)^4$

$= 1 + 12x + 54x^2 + 108x^3 + 81x^4$

b term in $x^2 = (1)(54x^2) + (4x)(12x) + (-x^2)(1)$

coefficient of $x^2 = 54 + 48 - 1 = 101$

13 **a** time $= 120 \times (0.9)^3 = 87.48$ seconds

b GP: $a = 120$, $r = 0.9$, $n = 12$

$S_{12} = \frac{120[1-(0.9)^{12}]}{1-0.9}$

$= 861.08$ seconds

$= 14$ mins 21 secs (nearest sec.)

14 $= [1+8(\frac{x}{2})+\frac{8\times7}{2}(\frac{x}{2})^2+\ldots][1+6(-x)+\frac{6\times5}{2}(-x)^2+\ldots]$

$= [1 + 4x + 7x^2 + \ldots][1 - 6x + 15x^2 + \ldots]$

$= 1 - 6x + 15x^2 + 4x - 24x^2 + 7x^2 + \ldots$

$= 1 - 2x - 2x^2 + \ldots$

$\therefore\; A = -2,\ B = -2$

15 **a** 6, 12, 24, 48

b GP: $a = 6$, $r = 2$, $n = 10$

$S_{10} = \frac{6(2^{10}-1)}{2-1} = 6138$

16 **a** $= 1 + 4x + 6x^2 + 4x^3 + x^4$

b $= 1 - 4x + 6x^2 - 4x^3 + x^4$

c $(1 + 4x + 6x^2 + 4x^3 + x^4)$

$+ (1 - 4x + 6x^2 - 4x^3 + x^4) = 82$

$2 + 12x^2 + 2x^4 = 82$

$x^4 + 6x^2 - 40 = 0$

$(x^2 + 10)(x^2 - 4) = 0$

$x^2 = -10$ [no real solutions] or $x^2 = 4$

$x = \pm 2$

17 **a** $a \times (1.5)^2 = 18$

$a = 18 \div 2.25 = 8$

b $S_6 = \frac{8[(1.5)^6 - 1]}{1.5-1} = 166.25$

c $8 \times (1.5)^{k-1} > 8000$

$(k-1)\lg 1.5 > \lg 1000$

$k > \frac{\lg 1000}{\lg 1.5} + 1$

$k > 18.04 \;\therefore$ smallest $k = 19$

18 $(1 + \frac{ax}{2})^{10} + (1 + bx)^{10}$

$= 1 + 10(\frac{ax}{2}) + \frac{10\times9}{2}(\frac{ax}{2})^2 + \ldots$

$+ 1 + 10(bx) + \frac{10\times9}{2}(bx)^2 + \ldots$

$= 2 + (5a + 10b)x + (\frac{45}{4}a^2 + 45b^2)x^2 + \ldots$

$\therefore \quad 5a + 10b = 0 \quad \Rightarrow \quad a = -2b$

and $\frac{45}{4}a^2 + 45b^2 = 90 \quad \Rightarrow \quad a^2 + 4b^2 = 8$

sub. $(-2b)^2 + 4b^2 = 8$

$b^2 = 1$

$a < b \;\therefore\; b = 1,\ a = -2$